REAL OPTION VALUE, CHAPTER 4, APPENDIX C

TWO FACTOR INVESTMENT INCENTIVES & SUBSIDIES[1]

1 Introduction

Do permanent or retractable government subsidies such as direct payments per unit revenue or per quantity produced, or specified feed-in-tariffs, or a renewable energy certificate or freedom from taxation, encourage early investment in renewable energy facilities? Does the size of the possible government subsidy reduce the price threshold that justifies investment significantly, when both unit prices and the units of production are stochastic, if the subsidy might be retracted?

The issue of the effect of government subsidies or charges on investment timing, when output prices are stochastic, is the original consideration in the first real option model of Tourinho (1979). Tourinho poses the dilemma that without a holding cost being imposed on the owner of an option to extract natural resources, the owner would never have a sufficient incentive to commit an irreversible investment to produce the resource. Other incentives to encourage early investment are the imposition (or presence) of an escalating investment cost, or as in Adkins and Paxson (2013) the existence of a convenience (or similar) yield for future prices of the underlying resource.

We use a Poisson (jump) process to model sudden provision of permanent or alternatively retractable subsidies. Several authors have incorporated jump processes into real investment

[1] Extracted from Adkins and Paxson (2015).

theory. Dixit and Pindyck (1994) discuss Poisson jump processes, and apply upward jumps to the expected capital gain from the possible implementation of an investment tax credit. Brach and Paxson (2003) consider Merton-style jumps in accounting for gene discovery and drug development failures and successes.

We consider that the instantaneous cash flow from a facility is the respective commodity price of the output times the quantity produced, and either there is no operating cost, or there is a fixed operating cost that can be incorporated into the investment cost. There are no other options embedded in the facility such as expansion, contraction, suspension or abandonment. Further assumptions are that the lifetime of the facility is infinite, there are no taxes or competition, and facility construction is instantaneous. Moreover, the typical assumptions of real options theory apply, with drifts, interest rates, convenience yields, volatilities and correlation constant over time, ignoring the seasonality and unreliability of prices and quantities.

We assume the primary government objective of subsidies is to reduce the private sector price threshold (keeping the quantity threshold constant) that justifies making an irreversible, instantaneous investment, instead of creating a high real option value for any allowable prospective facility or concession. Initially we ignore the possibility that such concessions might be purchased from (and thus benefit) the government.

Here is a broad menu of possible arrangements, that is some characteristic subsidies for such facilities, first where the subsidy is proportional to price times quantity, which is solved by simply scaling P*Q (Model I); then assuming there is a permanent subsidy proportional to the quantity generated (Model II); then assuming there is a retractable subsidy proportional to the quantity generated (Model III); and then assuming there is the possibility of a permanent subsidy proportional to Q (Model IV).

Then we compare the price thresholds and real option values using comparable base parameter values, and illustrate the sensitivity of these models to changes in some important variables such as quantity volatility, the subsidy rate and the intensities of possible sudden permanent or retractable subsidies.

2 Models

2.1 Model I Stochastic Price and Quantity

We consider a perpetual opportunity to construct a renewable energy facility, such as a hydro-electric plant or a wind farm or another renewable energy facility (biodiesel, ethanol), at a fixed investment cost K. This investment cost is treated as irreversible or irrecoverable once incurred. The value of this investment opportunity, denoted by F_1, depends on the amount of output sold per unit of time, denoted by Q, and the price per unit of output, denoted by P. Both of these variables are assumed to be stochastic and are assumed to follow geometric Brownian motion processes (gBm):

$$\mathrm{d}X = \alpha_X X\mathrm{d}t + \sigma_X X\mathrm{d}Z \tag{1}$$

for $X \in \{P, Q\}$, where α denotes the instantaneous drift parameter, σ the instantaneous volatility, and $\mathrm{d}Z$ the standard Wiener process. Potential correlation between the two variables is represented by ρ. Assuming risk neutrality and applying Ito's lemma, the partial differential equation (PDE) representing the value to invest for an inactive firm with an appropriate investment opportunity is:

$$\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_1}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_1}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_1}{\partial P \partial Q} + \theta_P P \frac{\partial F_1}{\partial P} + \theta_Q Q \frac{\partial F_1}{\partial Q} - rF_1 = 0. \qquad (2)$$

where θ_X denote the risk-neutral drift rates and r the risk-free rate, (θ=r-α). Following McDonald and Siegel (1986) and Adkins and Paxson (2011), the solution to (2) is:

$$F_1 = A_1 P^{\beta_1} Q^{\eta_1}. \qquad (3)$$

β_1 and η_1 are the power parameters for this option value function. Since there is an incentive to invest when both P and Q are sufficiently high but a disincentive when either are sufficiently low, we would expect both power parameter values to be positive. Also, the parameters are linked through the characteristic root equation found by substituting (3) in (2):

$$Q(\beta_1, \eta_1) = \tfrac{1}{2}\sigma_P^2 \beta_1 (\beta_1 - 1) + \tfrac{1}{2}\sigma_Q^2 \eta_1 (\eta_1 - 1) + \rho\sigma_P\sigma_Q \beta_1 \eta_1 + \theta_P \beta_1 + \theta_Q \eta_1 - r = 0. \qquad (4)$$

After the investment, the plant generates revenue equaling (1+τ)* PQ, where τ is the permanent subsidy proportional to the revenue sold (τ=0 indicates no possible subsidy). So from (2), the valuation relationship for the operational state is:

$$\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_1}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_1}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_1}{\partial P \partial Q} + \theta_P P \frac{\partial F_1}{\partial P} + \theta_Q Q \frac{\partial F_1}{\partial Q} + (1+\tau)PQ - rF_1 = 0, \quad (5)$$

After the investment (K), the solution to (5) is:

$$\frac{(1+\tau)PQ}{r - \mu_{PQ}},$$

where $\mu_{PQ} = \rho\sigma_P\sigma_Q + \theta_P + \theta_Q$, see Paxson and Pinto (2005). The investment is made when the two variables attain their respective thresholds. If we denote the threshold levels for P and Q by

$\hat{P}_1$ and $\hat{Q}_1$, respectively, and since value conservation requires the investment option value to be exactly balanced by the net value rendered by the investment, then the value matching relationship is specified by:

$$A\hat{P}_1^{\beta_1}\hat{Q}_1^{\eta_1} = \frac{(1+\tau)\hat{P}_1\hat{Q}_1}{r-\mu_{PQ}} - K \, . \tag{6}$$

Optimality is characterized by the two smooth pasting conditions associated with (6) for P and Q, respectively:

$$\beta_1 A\hat{P}_1^{\beta_1}\hat{Q}_1^{\eta_1} = \frac{(1+\tau)\hat{P}_1\hat{Q}_1}{r-\mu_{PQ}} \, , \tag{7}$$

$$\eta_1 A\hat{P}_1^{\beta_1}\hat{Q}_1^{\eta_1} = \frac{(1+\tau)\hat{P}_1\hat{Q}_1}{r-\mu_{PQ}} \, . \tag{8}$$

From (7) and (8), our conjecture that the parameter values are positive is corroborated because of the non-negativity of the investment option value. Moreover, the parameters are equal, $\beta_1 = \eta_1$. This establishes that for determining the optimal investment policy, the two factors can be simply represented by their product PQ, the revenue from generating output per unit of time. This substitution is originally proposed by Paxson and Pinto (2005), who apply the principle of similarity for reducing the dimension of (5) to one in order to obtain a closed-form solution. It follows that:

$$\frac{(1+\tau)\hat{P}_1\hat{Q}_1}{r-\mu_{PQ}} = \frac{\beta_1}{\beta_1 - 1} K \, , \tag{9}$$

where β_1 is determined from $Q(\beta_1, \beta_1) = 0$, (4). Also

$$F_1 = \begin{cases} A_1 P_1^{\beta_1} Q_1^{\eta_1} \quad \text{for} \quad PQ < \hat{P}_1 \hat{Q}_1, \\ \dfrac{(1+\tau) P_1 Q_1}{r - \mu_{PQ}} - K \quad \text{for} \quad PQ \geq \hat{P}_1 \hat{Q}_1. \end{cases} \tag{10}$$

with:

$$A_1 = \frac{(1+\tau)\hat{P}_1^{1-\beta_1} \hat{Q}_1^{1-\eta_1}}{\eta_1 (r - \mu_{PQ})}.$$

2.2 Model II

Stochastic Price and Quantity with a Permanent Subsidy on Quantity

We now modify the analysis to consider the impact on the investment decision of a permanent government subsidy, denoted by τ, whose value is proportional to the amount of output Q sold per unit of time. In the presence of the subsidy, the generating plant is effectively producing two distinct outputs: (i) the revenue per unit of time generated by the plant PQ, and (ii) the subsidy revenue received from the government or power customers τQ. As before, the investment option value denoted by F_2 depends on the two factors P and Q. The risk neutral valuation relationship for F_2 takes a similar form as (2), so the valuation function is given by (3) except for the change in subscript, that is $F_2 = A_2 P^{\beta_2} Q^{\eta_2}$. Also, its characteristic root equation is $Q(\beta_2, \eta_2) = 0$, (4).

After incurring the investment, the present value of the operating revenue for the plant is:

$$\frac{PQ}{r - \mu_{PQ}} + \frac{\tau Q}{r - \theta_Q}.$$

The operating revenue is the present value of the operating revenue plus the government subsidy. If the two threshold levels signaling optimal investment are denoted by $\hat{P}_2$ and $\hat{Q}_2$ for P and Q, respectively, then the value matching relationship for this subsidized production model is:

$$A_2 \hat{P}_2^{\beta_2} \hat{Q}_2^{\eta_2} = \frac{\hat{P}_2 \hat{Q}_2}{r - \mu_{PQ}} + \frac{\tau \hat{Q}_2}{r - \theta_Q} - K \, . \tag{11}$$

It is observable from (11) that the principle of similarity is no longer available, since the factors P and Q occurring in the relationship cannot be construed as a product PQ, even if $\beta_2 = \eta_2$. The two smooth pasting conditions associated with (11) are:

$$\beta_2 A_2 \hat{P}_2^{\beta_2} \hat{Q}_2^{\eta_2} = \frac{\hat{P}_2 \hat{Q}_2}{r - \mu_{PQ}} \, , \tag{12}$$

$$\eta_2 A_2 \hat{P}_2^{\beta_2} \hat{Q}_2^{\eta_2} = \frac{\hat{P}_2 \hat{Q}_2}{r - \mu_{PQ}} + \frac{\tau \hat{Q}_2}{r - \theta_Q} \, . \tag{13}$$

These conditions, (12) and (13), reveal that both β_2 and η_2 are positive, otherwise the option value at investment $A_2 \hat{P}_2^{\beta_2} \hat{Q}_2^{\eta_2}$ would be negative. We obtain reduced form value matching relationships by substituting (12) and (13) in (11), respectively:

$$\frac{\hat{P}_2 \hat{Q}_2}{r - \mu_{PQ}} = \frac{\beta_2}{\beta_2 - 1} \left(K - \frac{\tau \hat{Q}_2}{r - \theta_Q} \right) , \tag{14}$$

$$\frac{\hat{P}_2 \hat{Q}_2}{r - \mu_{PQ}} = \frac{\eta_2}{\eta_2 - 1} K - \frac{\tau \hat{Q}_2}{r - \theta_Q} \, . \tag{15}$$

In these reduced forms, the government subsidy effectively reduces the effective investment cost of the plant with the economic consequence that the optimal revenue threshold justifying the investment is lower than without it.

The thresholds for the output sold per unit of time Q and the price per unit of output P economically justifying an optimal investment are specified by the two reduced form value matching relationships, (14) and (15), and (4) the characteristic root equation $Q(\beta_2, \eta_2) = 0$. In principle, the boundary relationship is obtainable by eliminating β_2 and η_2 from the three constituent equations, but as no purely analytical solution exists, we resort to obtaining the boundary numerically, solving sets of equations simultaneously.

2.3 Model III

Stochastic Price and Quantity with a Retractable Subsidy on Quantity

Subsidies are normally offered by governments in order to induce entrepreneurs to accelerate the timing of their investment in facilities, when otherwise they would defer making their commitment. As soon as the subsidy has activated sufficient plant investment, the government may decide to withdraw the subsidy, often without any advance warning. We assume that once the subsidy is withdrawn, it will never again be provided.

We denote the value of the investment option in the presence of a subsidy, but when there is a possibility of an immediate withdrawal, by F_3, and in the absence of a subsidy by F_1 (when τ=0), as before. We assume that the subsidy withdrawal is well explained by a Poisson process with a constant intensity factor, denoted by λ. The change in the option value conditional on the subsidy withdrawal occurring is $F_1(P,Q) - F_3(P,Q)$, so the expected change is given by:

$$\{F_1(P,Q) - F_3(P,Q)\}\lambda \mathrm{d}t + \{0\}(1 - \lambda \mathrm{d}t) = \lambda\{F_1(P,Q) - F_3(P,Q)\}\mathrm{d}t \,.$$

From (2), it follows that the risk-neutral valuation relationship for F_3 is:

$$\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_3}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_3}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_3}{\partial P \partial Q} + \theta_P P \frac{\partial F_3}{\partial P} + \theta_Q Q \frac{\partial F_3}{\partial Q} + \lambda F_1 - (r+\lambda) F_3 = 0. \tag{16}$$

The solution to (16) adopts the form:

$$F_3 = A_3 P^{\beta_3} Q^{\eta_3} + A_1 P^{\beta_1} Q^{\eta_1}, \tag{17}$$

where the parameters β_1 and η_1 are specified by $Q(\beta_1, \eta_1) = 0$, (4), with $\beta_1 = \eta_1$ (with τ=0), while β_3 and η_3 are related through the characteristic root equation:

$$Q_3(\beta_3, \eta_3) = \tfrac{1}{2}\sigma_P^2 \beta_3(\beta_3 - 1) + \tfrac{1}{2}\sigma_Q^2 \eta_3(\eta_3 - 1) + \rho\sigma_P\sigma_Q\beta_3\eta_3 + \theta_P\beta_3 + \theta_Q\eta_3 - (r+\lambda) = 0. \tag{18}$$

For any feasible values of P and Q, the valuation function F_3 exceeds F_1 because the coefficient A_3 is positive. This implies that the option value to invest is always greater in the presence of a government subsidy that may be withdrawn unexpectedly than in its absence, which suggests that a subsidy, even one having an unexpected withdrawal, comparatively hastens the investment commitment.

If the subsidy is present, then the present value of the plant is $PQ/(r-\mu_{PQ}) + \tau Q/(r-\theta_Q)$, and if absent, then $PQ/(r-\mu_{PQ})$, so the net present value after the investment is:

$$\frac{PQ}{r-\mu_{PQ}} + \frac{(1-\lambda)\tau Q}{r-\theta_Q}.$$

The thresholds signaling investment for a subsidy with unexpected withdrawal are denoted by $\hat{P}_3$ and $\hat{Q}_3$ for P and Q, respectively. The value matching condition becomes:

$$A_3\hat{P}_3^{\beta_3}\hat{Q}_3^{\eta_3} + A_1\hat{P}_3^{\beta_1}\hat{Q}_3^{\eta_1} = \frac{\hat{P}_3\hat{Q}_3}{r-\mu_{PQ}} + \frac{(1-\lambda)\tau\hat{Q}_3}{r-\theta_Q} - K \,. \tag{19}$$

The two associated smooth pasting conditions are, respectively:

$$\beta_3 A_3\hat{P}_3^{\beta_3}\hat{Q}_3^{\eta_3} + \beta_1 A_1\hat{P}_3^{\beta_1}\hat{Q}_3^{\eta_1} = \frac{\hat{P}_3\hat{Q}_3}{r-\mu_{PQ}} \,, \tag{20}$$

$$\eta_3 A_3\hat{P}_3^{\beta_3}\hat{Q}_3^{\eta_3} + \eta_1 A_1\hat{P}_3^{\beta_1}\hat{Q}_3^{\eta_1} = \frac{\hat{P}_3\hat{Q}_3}{r-\mu_{PQ}} + \frac{(1-\lambda)\tau\hat{Q}_3}{r-\theta_Q} \,. \tag{21}$$

The parameter values A_1, β_1 and η_1 are known from the solution to Model I with τ=0.

$$A_3 = (-\beta_1 A_1\hat{P}_3^{\beta_1}\hat{Q}_3^{\eta_1} + \frac{\hat{P}_3\hat{Q}_3}{r-\mu_{PQ}}) / (\beta_3\hat{P}_3^{\beta_3}\hat{Q}_3^{\eta_3})$$

2.4 Model IV Stochastic Joint Products with Sudden Provision of a Permanent Subsidy on Quantities

We now explore the financial consequences on the investment decision for a subsidy that can be provided permanently at any time, in order to determine its effects on the threshold levels for P and Q. We consider now only the case where the subsidy thereafter can never be withdrawn, and compare the case of building the facility without a possible subsidy with the cases of a permanent subsidy.

Since a sudden unexpected subsidy withdrawal makes an operating plant appear to be less economically attractive, it is likely that investment is hastened to capture the subsidy before it is withdrawn. In contrast, a sudden unexpected permanent subsidy introduction is expected to produce the opposite effect of investment deferral so that the subsidy income can be more fully captured.

In Model II, the revenue threshold that signals an economically justified investment in the presence of a subsidy is $\hat{R}_2 = \hat{P}_2\hat{Q}_2$. Before the investment is made, the threshold $\hat{R}_2$ creates either side separate domains over which the investment option value differs in form. The prevailing revenue is denoted by $R = PQ$. If the prevailing revenue R is less than the threshold $\hat{R}_2$, then a sudden unexpected subsidy announcement does not trigger an immediate investment and the investment is deferred until R attains $\hat{R}_2$. If, on the other hand, $R \geq \hat{R}_2$, then a sudden unexpected subsidy announcement automatically triggers an immediate investment in the plant. This asymmetry around the threshold $\hat{R}_2$ means that the investigation of a sudden unexpected subsidy announcement has to treat the case where $R < \hat{R}_2$ differently from where $R \geq \hat{R}_2$.

The value for the investment option, denoted by F_4, is specified over the two domains:

$$F_4 = \begin{cases} F_{40} & \text{for } R < \hat{R}_2, \\ F_{41} & \text{for } R \geq \hat{R}_2. \end{cases} \tag{22}$$

Here we only consider the domain $R < \hat{R}_2$, which is considered to be below threshold because over this domain, in the presence of a subsidy investment is not economically justified. It is assumed that a subsidy introduction is well described by a Poisson process with intensity λ, and that once introduced, it cannot be withdrawn. The risk neutral valuation relationship then becomes:

$$\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_{40}}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_{40}}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_{40}}{\partial P \partial Q} + \theta_P P \frac{\partial F_{40}}{\partial P} + \theta_Q Q \frac{\partial F_{40}}{\partial Q} + \lambda F_2 - (r+\lambda) F_{40} = 0. \tag{23}$$

The solution to (23) adopts the form:

$$F_{40} = A_{40} P^{\beta_{40}} Q^{\eta_{40}} + A_2 P^{\beta_2} Q^{\eta_2} \tag{24}$$

where the parameters β_2 and η_2 are specified by $Q(\beta_2, \eta_2) = 0$, (4), and β_{40} and η_{40} by $Q_3(\beta_{40}, \eta_{40}) = 0$, (18).

If there is no subsidy, then the present value of the plant is given by $PQ/(r-\mu_{PQ})$, while if there is an additional subsidy, then the present value is $PQ/(r-\mu_{PQ}) + \tau Q/(r-\theta_Q)$. The net operating present value after the investment is given by:

$$\frac{PQ}{r-\mu_{PQ}} + \frac{\lambda \tau Q}{r-\theta_Q}.$$

The thresholds signaling investment for a sudden unexpected subsidy introduction are denoted by $\hat{P}_{40}$ and $\hat{Q}_{40}$ for P and Q, respectively. The value matching condition becomes:

$$A_{40}\hat{P}_{40}^{\beta_{40}}\hat{Q}_{40}^{\eta_{40}} + A_2\hat{P}_{40}^{\beta_2}\hat{Q}_{40}^{\eta_2} = \frac{\hat{P}_{40}\hat{Q}_{40}}{r-\mu_{PQ}} + \frac{\lambda\tau\hat{Q}_{40}}{r-\theta_Q} - K . \tag{25}$$

The two associated smooth pasting conditions can be expressed as, respectively:

$$\beta_{40}A_{40}\hat{P}_{40}^{\beta_{40}}\hat{Q}_{40}^{\eta_{40}} + \beta_2 A_2\hat{P}_{40}^{\beta_2}\hat{Q}_{40}^{\eta_2} = \frac{\hat{P}_{40}\hat{Q}_{40}}{r-\mu_{PQ}}, \tag{26}$$

$$\eta_{40}A_{40}\hat{P}_{40}^{\beta_{40}}\hat{Q}_{40}^{\eta_{40}} + \eta_2 A_2\hat{P}_{40}^{\beta_2}\hat{Q}_{40}^{\eta_2} = \frac{\hat{P}_{40}\hat{Q}_{40}}{r-\mu_{PQ}} + \frac{\lambda\tau\hat{Q}_{40}}{r-\theta_Q}. \tag{27}$$

The effect of an unexpected sudden subsidy introduction is to raise the effective investment cost by an amount equaling the option value for an economically justified investment in the presence of a subsidy, adjusted by the Poisson intensity parameter λ.

3. Numerical Illustrations

It is interesting to compare the apparent effectiveness of different subsidy arrangements, and the possible sudden introduction or retraction of those subsidies on the real option value of those investment opportunities, and the price and quantity thresholds that justify commencing investments. Pairs of $\hat{P}$ and $\hat{Q}$ could be generated by changing the solutions along a suitable Q range. Since Model I $\hat{P}$ with τ=.20 is less than Model I $\hat{P}$ without a subsidy (τ=0), clearly a permanent subsidy makes a difference, with a 20% R subsidy reducing the price threshold by 16.6%, and increasing the ROV almost 60%, as shown in Table I..

Table I

Subsidy Incentive Effect under Different Models								
	τ	P^Q^	P^ (Q^=7.8)	ROV				
Model I	0.00	638.70	81.88	1022.72	NO SUBSIDY			
Model I	0.20	532.25	68.24	1631.49	PERMANENT SUBSIDY ON R			
Model II	13.65	486.07	62.32	**1903.76**	PERMANENT SUBSIDY ON Q			
Model III	13.65	461.72	59.19	1717.11	RETRACTABLE SUBSIDY ON Q			
Model IV0	13.65	718.97	92.18	1325.66	MAYBE PERMANENT SUBSIDY ON Q, R<R2^			
Q^=Q			7.80					
P			53.00					
R			413.4					
R Subsidy			106.45		Subsidy Value at R^			M I
Q Subsidy			106.47		Subsidy Value at P^Q^			M II

Model I is the solution to EQs 6,-7-8 with ROV EQ 10, Model II is the solution to EQs 11-12-13 with ROV EQ 3 amended, Model III is the solution to EQs 19-20-21 with ROV EQ 17, Model IV is the solution to EQs 25-26-27 with ROV EQ 24, all with also the Q function, either EQ 4 or 18, with the parameter values as follows: price P=€53, quantity Q=7.8 KWh, R subsidy τ=.20, Q subsidy 13.65, investment cost K=€4,867,000[2], price volatility σ_P=.20, quantity volatility σ_Q=.20, price and quantity correlation ρ=-.50, θ_P=.01, θ_Q=.01, and riskless interest rate r=.08. λ=.10 reflects the possibility of a subsidy being withdrawn, and both the possibility of a permanent subsidy and also a retractable subsidy. P^Q^ indicates the total revenue, P^ indicates the P threshold that justifies commencing the investment, given that Q^=7.8.

For a comparable subsidy (at the price threshold) on the quantity generated, Model II, the permanent subsidy reduces the price threshold even more, and adds more than 16% to the ROV. R is more uncertain (34.6%) than Q due to the assumed volatilities and negative correlation.

Permanent versus Retractable Subsidies

The lowest price threshold given $\hat{Q}$ =7.8 is indicated in bold red, retractable Model III. At P = € 53 the highest ROV indicated in bold is Model II, the permanent subsidy, which would not provide the greatest incentive to commence investment. Commence the project when the subsidy is available earlier if it might be withdrawn, a “flighty bird in hand”.

A higher retractable λ results in $\hat{P}$ increasing slightly and ROV decreasing, as shown in Figure 5 below. Comparing the below threshold Model IV0 (maybe permanent) with the below threshold Model III (retractable), the P_{IV0} price threshold exceeds P_{III}, naturally because a bird in the hand is worth more than the same bird in a bush (talk is cheap), and the ROV is lower. The most valuable ROV is Model II, which is the permanent subsidy. Governments seeking to sell concessions for the ROV might contemplate permanent subsidies, but charging immediately for the subsidies as a real option.

SENSITIVITIES

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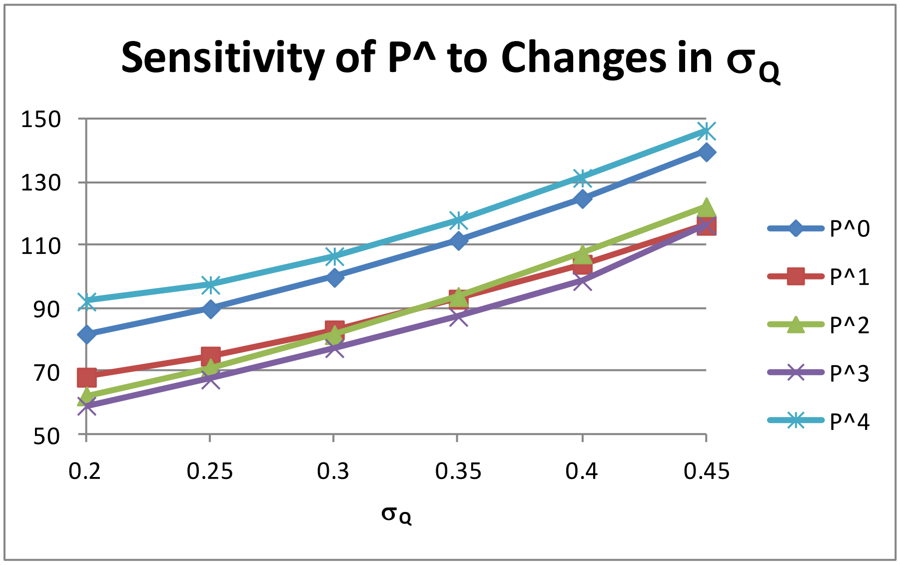
[2] Some of the P, Q and K parameter values are consistent with an Iberian wind farm with a capacity of 3MW operating at average load factor of around 30%. The subsidy rate .20 for R in Model I is comparable with the Q=13.65 subsidy in Model II at the P,Q which justifies exercise of the real option.

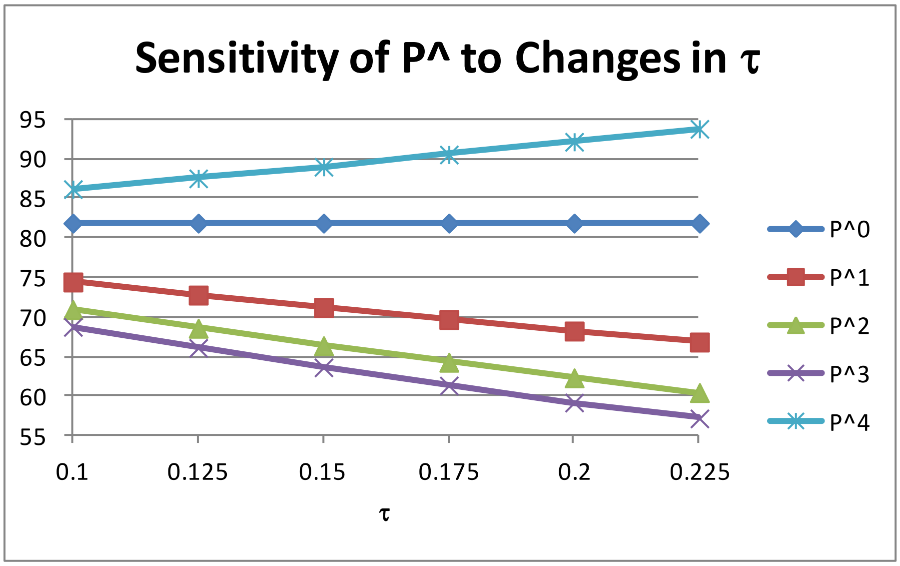
Our base parameters for the sensitivity of $\hat{P}$ and ROV to changes in parameter values are the same as for Table I, over a range of Q volatility 20% to 45%, τ from .10 to .225 (and the comparables for Q), and λ from .09 to .115, both for retractable, and for possible permanent subsidies. Figure 1



$\hat{P}_0$ is the solution to EQs 6-7-8 without a subsidy, and $\hat{P}_1$ with a subsidy, $\hat{P}_2$ is the solution to EQs 11-12-13, $\hat{P}_3$ is the solution to EQs 19-20-21, $\hat{P}_4$ is the solution to EQs 25-26-27 with the parameter values in Table I.

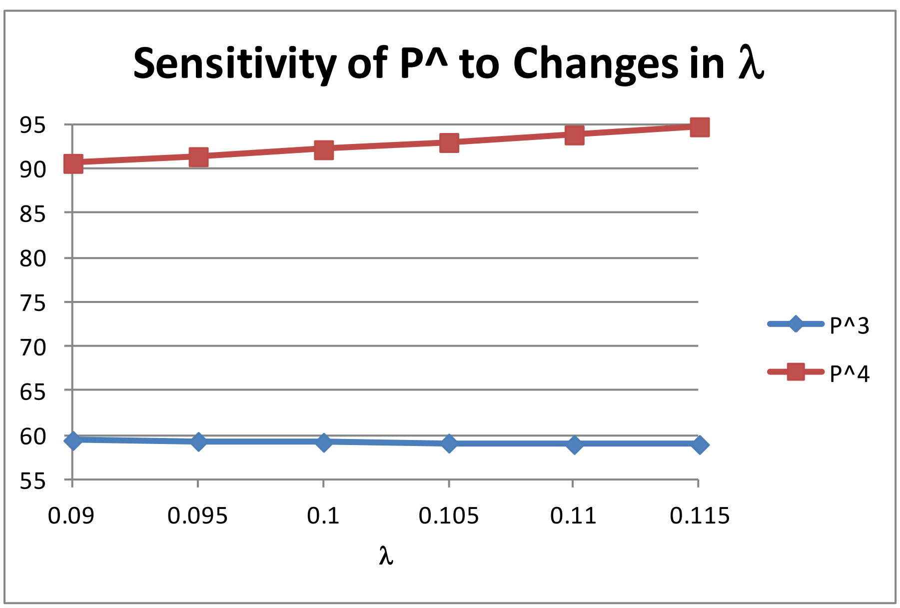
Price thresholds for the first four models increase with increases with quantity volatility Figure 1. Figure 2 shows that naturally Model 0 is not affected by changes in the size of the subsidy, but otherwise except for Model IV, increasing the subsidy provides a positive incentive for early investment. But for Model IV , increasing the size of a possible permanent subsidy may delay early investment. So either production volume floors (quotas) or high actual permanent or retractable subsidies might encourage early investment. Sensitivity to the probability of subsidies is shown in Figure 3. Generally the higher the probability of a possible permanent subsidy results in an incentive to delay early investment.

Figure 2



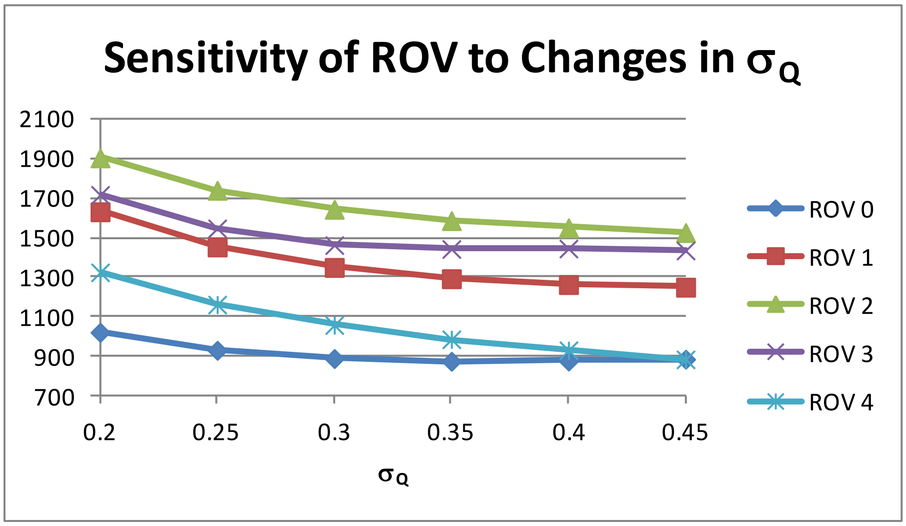
$\hat{P}_0$ is the solution to EQs 6-7-8 without a subsidy, and $\hat{P}_1$ with a subsidy, $\hat{P}_2$ is the solution to EQs 11-12-13, $\hat{P}_3$ is the solution to EQs 19-20-21, $\hat{P}_4$ is the solution to EQs 25-26-27 with the parameter values in Table I.

Figure 3



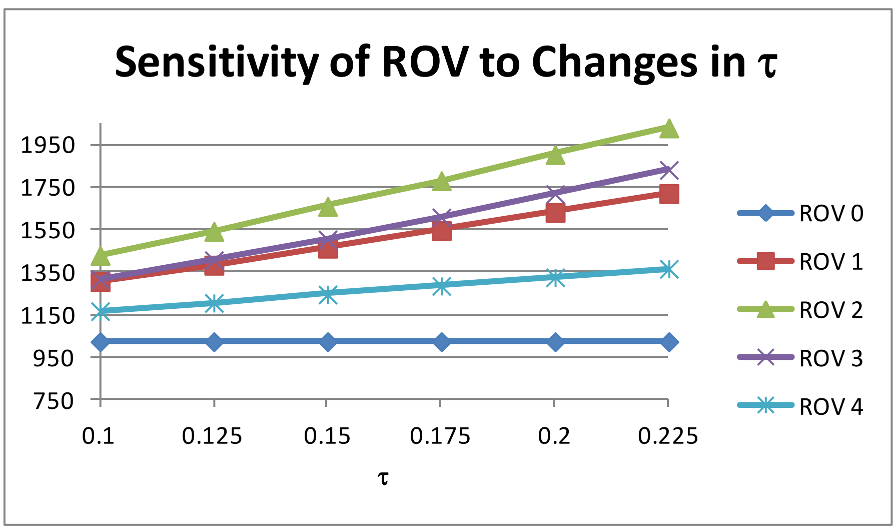
$\hat{P}_3$ is the solution to EQs 19-20-21, $\hat{P}_4$ is the solution to EQs 25-26-27 with the parameter values in Table I.

Figure 4



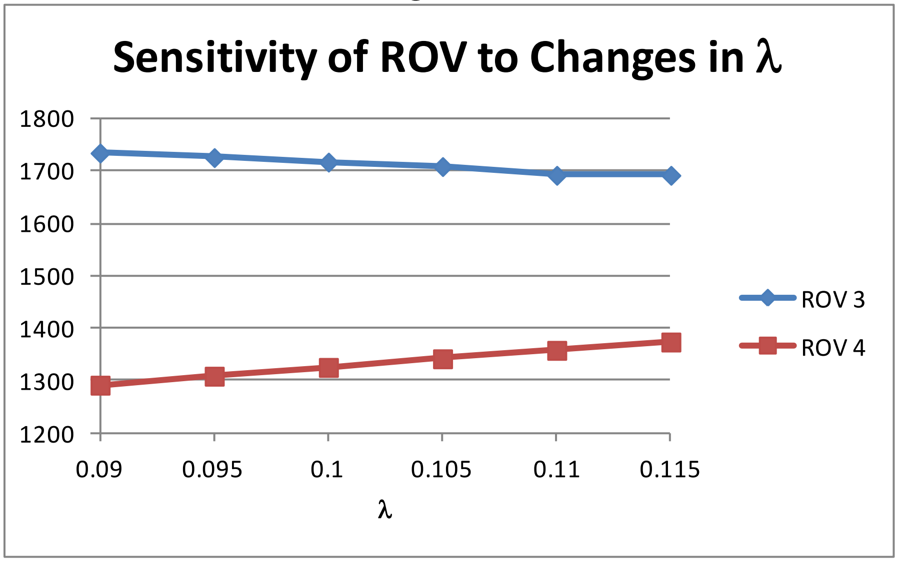
ROV_0 is the solution to EQ 10 without a subsidy, ROV_1 with a subsidy, ROV_2 the LHS of EQ 11, ROV_3 EQ 17, ROV_4 EQ 24 with the parameter values in Table I.

Figure 5



ROV_0 is the solution to EQ 10 without a subsidy, ROV_1 with a subsidy, ROV_2 EQ 3 amended, ROV_3 EQ 17, ROV_4 EQ 24 with the parameter values in Table I.

Figure 6



ROV_3 is the solution to EQ 17, ROV_4 EQ 24 with the parameter values in Table I.

The ROV for nearly all models decrease with increases of quantity volatility (when there is negative correlation) . This is due to the negative correlation with P acting as a kind of natural hedge against Q, resulting in lower overall volatility. So while either production volume floors or high subsidies of any type might encourage investment, the value of a renewable energy concession will be dependent on expected volatilities, as well as the subsidy.

Sensitivity of ROV to possible retraction is intuitive. The greater the probability of retracting a subsidy, the lower the ROV, Model III. Of course, the greater the possibility of a permanent subsidy, Model IV, the greater the ROV.

In summary, a chief real options manager primarily interested in ROV before investment if P is below threshold, will seek permanent subsidies (Model II) or retractable subsidies (Model III), particularly if the concession is free, rather than purchased at the ROV from the government. A government seeking early investments, thus low price thresholds, will favor arrangements given by Model III, unless full value for granting the concession can be realized. In that case, there is a trade-off between the present value of subsidies and the current value of the concession.

What are the apparent policy guidelines in using subsidies to encourage early investment in facilities with joint (and sometimes distinct) products? Subsidies matter, especially if regarded as permanent. But whether increasing a subsidy say from .10 to .225 R (or equivalent) is worth reducing the threshold as indicated is questionable. Possibly less transparent incentives are price or quantity guarantees, which effectively reduce price and/or quantity volatility, with a significant impact on thresholds under all models.

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	A	B	C
1	**JOINT PRODUCTS MODEL I**		
2	INPUT	Stochastic P & Q	
3	P	53.00	
4	Q	7.80	
5	R	413.40	
6	K	4867.00	
7	σ_P	0.20	
8	σ_Q	**0.20**	
9	ρ	-0.50	
10	r	0.08	
11	θ_P	0.01	
12	θ_Q	0.01	
13	μ_{PQ}	0.0000	B9*B7*B8+B11+B12
14	$r-\mu_{PQ}$	0.0800	
15	τ	0.2000	SUBSIDY ON R
16	OUTPUT		
17	F1(P,Q)	1631.49	IF(B5<B28, B23*(B3^B24)*(B4^B25),B18)
18	F1(P,Q)	1334.00	(1+B15)*B5/B14-B6
19	VM	0.0000	B23*(B26^B24)*(B27^B25)-((1+B15)*(B26*B27)/B14)+B6
20	SP1	0.0000	B24*B23*(B26^(B24-1))*(B27^B25)-(1+B15)*B27/B14
21	SP2	0.0000	B25*B23*(B26^B24)*(B27^(B25-1))-B26/B14
22	Q	0.0000	0.5*(B7^2)*B24*(B24-1)+0.5*(B8^2)*B25*(B25-1)+B9*B7*B8*B24*B25+B24*B11+B25*B12-B10
23	A1	0.0003	
24	β_1	2.56	
25	η_1	2.56	
26	P^	68.24	
27	Q^	7.80	SPECIFIED
28	P^ Q^	532.25	
29	SOLVER	0.0000	SET B29=0, CHANGING B23:B26

	A	B	C
1	**JOINT PRODUCTS MODEL II**		
2	INPUT	Stochastic P & Q	
3	P	53.00	
4	Q	7.80	
5	R	413.40	
6	K	4867.00	
7	σ_P	0.20	
8	σ_Q	**0.20**	
9	ρ	-0.50	
10	r	0.08	
11	θ_P	0.01	
12	θ_Q	0.01	
13	μ_{PQ}	0.0000	
14	$r-\mu_{PQ}$	0.0800	
15	τ	13.6500	SUBSIDY ON Q
16			
17	F2(P,Q)	1903.76	IF(B5<B28, B23*(B3^B24)*(B4^B25),B18)
18	F2(P,Q)	1821.50	B5/B14+(B15*B4)/(B10-B12)-B6
19	VM1	0.0000	(B26*B27)/B14+B15*B27/(B10-B12)-B6-B23*(B26^B24)*(B27^B25)
20	SP1	0.0000	(B26*B27)/B14-B24*B23*(B26^B24)*(B27^B25)
21	SP2	0.0000	(B26*B27)/B14+9B15*B27/(B10-B12))-B25*B23*(B26^B24)*(B27^B25)
22	Q	0.0000	0.5*(B7^2)*B24*(B24-1)+0.5*(B8^2)*B25*(B25-1)+B9*B7*B8*B24*B25+B24*B11+B25*B12-B10
23	A2	0.0009	
24	β_1	2.23	
25	η_1	2.78	
26	P^	62.32	
27	Q^	7.80	
28	P^ Q^	486.07	
29	SOLVER	0.0000	SET B29=0, CHANGING B23:B26

	A	B	C
1			**JOINT PRODUCTS MODEL III**
2	INPUT	Stochastic P & Q	
3	P	53.00	
4	Q	7.80	
5	R	413.40	
6	K	4867.00	
7	σ_P	0.20	
8	σ_Q	**0.20**	
9	ρ	-0.50	
10	r	0.08	
11	θ_P	0.01	
12	θ_Q	0.01	
13	μ_{PQ}	0.0000	
14	$r-\mu_{PQ}$	0.0800	
15	τ	13.6500	SUBSIDY ON Q
16	λ	0.1000	POSSIBILITY OF SUBSIDY WITHDRAWAL
17	F3(P,Q)	1717.11	IF(B5<B28, B23*(B3^B24)*(B4^B25)+B30*(B3^B31)*(B4^B32),B18)
18	F3(P,Q)	1669.40	B5/B14+(1-B16)*(B15*B4)/(B10-B12)-B6
19	VM	0.0000	(B26*B27)/B14+(1-B16)*B15*B27/(B10-B12)-B6-B23*(B26^B24)*(B27^B25)-B30*(B26^B31)*(B27^B32)
20	SP1	0.0000	(B26*B27)/B14-B24*B23*(B26^B24)*(B27^B25)-B31*B30*(B26^B31)*(B27^B32)
21	SP2	0.0000	(B26*B27)/B14+((1-B16)*B15*B27/(B10-B12))-B25*B23*(B26^B24)*(B27^B25)-B32*B30*(B26^B31)*(B27^B32)
22	Q	0.0000	0.5*(B7^2)*B24*(B24-1)+0.5*(B8^2)*B25*(B25-1)+B9*B7*B8*B24*B25+B24*B11+B25*B12-(B10+B16)
23	A3	0.000009	
24	β_3	2.50	
25	η_3	4.00	
26	P^3	59.19	
27	Q^3	7.80	
28	P^ Q^	461.72	
29	SOLVER	0.0000	SET B29=0, CHANGING B23:B26
30	A1	0.00	
31	β_1	2.56	
32	η_1	2.56	
33	P^	81.88	M1: No subsidy
34	Q^	7.80	
35	P^ Q^	638.70	
36	SUBSIDY	106.47	
37	SUBSIDY	106.45	

	A	B	C
1			**JOINT PRODUCTS MODEL IV0**
2	INPUT	Stochastic P & Q	
3	P	53.00	
4	Q	7.80	
5	R	413.40	R<R2
6	K	4867.00	
7	σ_P	0.20	
8	σ_Q	**0.20**	
9	ρ	-0.50	
10	r	0.08	
11	θ_P	0.01	
12	θ_Q	0.01	
13	μ_{PQ}	0.0000	
14	$r-\mu_{PQ}$	0.0800	
15	τ	13.6500	SUBSIDY ON Q
16	λ	0.1000	POSSIBILITY OF SUDDEN SUBSIDY
17	F40(P,Q)	1325.66	IF(B5<B28, B23*(B3^B24)*(B4^B25)+B30*(B3^B31)*(B4^B32),B18)
18	F41(P,Q)	452.60	(B5/B14)+(B16*B15*B4)/(B10-B12)-B6
19	VM	0.0000	(B26*B27)/B14+(B16*B15*B27)/(B10-B12)-B6-B23*(B26^B24)*(B27^B25)-B30*(B26^B31)*(B27^B32)
20	SP1	0.0000	(B26*B27)/B14-B24*B23*(B26^B24)*(B27^B25)-B31*B30*(B26^B31)*(B27^B32)
21	SP2	0.0000	(B26*B27)/B14+(B16*B15*B27)/(B10-B12)-B25*B23*(B26^B24)*(B27^B25)-B32*B30*(B26^B31)*(B27^B32)
22	Q	0.0000	0.5*(B7^2)*B24*(B24-1)+0.5*(B8^2)*B25*(B25-1)+B9*B7*B8*B24*B25+B24*B11+B25*B12-(B10+B16)
23	A40	-0.0000090	
24	β40	2.46	
25	η40	4.00	
26	P^40	92.18	
27	Q^40	7.80	
28	P^ Q^40	718.97	
29	SOLVER	0.00	SET B29=0, CHANGING B23:B26
30	A2	0.000911	
31	β2	2.23	
32	η2	2.78	
33	P^	62.32	
34	Q^	7.80	
35	P2^Q2^	486.07	